MODIFICATION OF THE THEORY OF PLASTIC FLOW BASED ON A SHEAR-STRAIN MECHANISM

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UDC 539.374

The governing relationships of theories of flow type depend in a substantial manner on the selection of two functions, the loading function and the hardening function, whose detailed construction is not elucidated. The allowable freedom in giving these functions permits the assumption of sufficiently diverse loading surface shapes of both regular and singular type.

Results of experimental investigations available at this time do not reflect sufficiently completely the viewpoint of classical representations in the theory of a hardening plastic body. For instance, loadings with a constant stress intensity result in a significant growth in the plastic strain [1], indicating an inconsistency in the classical flow law based on an isotropically broadening Mises flow surface.

An experimentally based modification of the theory of plastic flow is proposed in this paper, in which a macroscopic shear strain mechanism, which is a particular case of a mechanical model of a material [2] and relies on experimental observations beyond the Luders lines, is taken as the basis for constructing the governing relationships. Such an approach does not use the concept of a loading surface to construct the governing relationships, but allows interpretation in this terminology. The loading surface is singular in the hardening stage, and comprised of piecewise-smooth sections of surfaces of constant principal tangential stresses. The appearance of plastic strain is associated with the Tresk–Saint-Venant plasticity condition, and hardening is developed as follows: the loading point in stress space displaces piecewise-smooth sections of the surfaces of constant principal tangential stresses parallel to themselves by withdrawing them from the origin.

It should be noted that the proposed modification of flow theory can be obtained formally from the assumption of orthotropy of the plastic state in the form [3] for a definite selection of the orthotropy coefficients [4].

1. During material loading, let a homogeneous stress-strain state be achieved. We denote the principal normal stresses at the time of plastic strain occurrence by σ_i ($i = 1, 2, 3$), where we agree to number the principal axes so that

$$\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3. \tag{1.1}$$

We introduce the following notation for the principal tangential stresses

$$T = (\sigma_1 - \sigma_3)/2,\ T_{12} = (\sigma_1 - \sigma_2)/2,\ T_{23} = (\sigma_2 - \sigma_3)/2,$$

then it follows from (1.1) that $T > 0$, $T_{12} \geqslant 0$, $T_{23} \geqslant 0$.

Later the strains are considered small and are represented in the form of the sum of elastic and plastic components. The plastic components of the principal elongations will be denoted by e_i ($i = 1, 2, 3$) and the principal plastic shear by $\gamma = e_1 - e_3$.

Let us also take the condition of plastic incompressibility and Hooke's law between the increment of the elastic strain component and the stress increment. It is considered that the elastic properties of the material do not change during the plastic deformation.

Let τ_s be the yield point under pure torsion, when $2T_{12} = 2T_{23} = T$.

The cases $T > \tau_s$, T_{12} and $T_{23} < \tau_s$ will be called the state of incomplete plasticity, and when T and $T_{12} > \tau_s$, $T_{23} < \tau_s$ (or T and $T_{23} > \tau_s$, $T_{12} < \tau_s$) the state of complete plasticity.

Mechanical Model of a Material. We will consider the material in the plastic state to be weakened only in the directions of the slip system (the directions of principal tangential

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 133-138, November-December, 1982. Original article submitted October 29, 1981.

 0021-8944/82/2306-0860$07.50

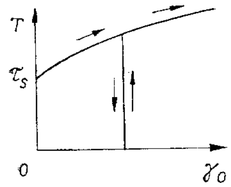

Fig. 1

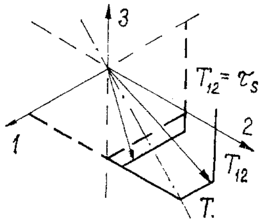

Fig. 2

stress action in which the limit τ_s is exceeded) and the plastic strain increment is a sequence of simple shears originating under the action of the growth of the intrinsic tangential stresses in these directions. The dependence of the principal plastic shear γ_0 on the magnitude of the maximum tangential stress during torsion (Fig. 1) is taken as the technical specification of the material in each of the slip systems.

We introduce the plastic hardening modulus $G_0(T) = \Delta T/\Delta\gamma_0(T)$.

For the incomplete plasticity state, the magnitude of simple shear in the direction T is determined by the dependence $\Delta\gamma^\nu = ZT/G_0$, where Δ denotes the change in the corresponding quantity.

In the principal stress axes we obtain the relationships

$$\Delta e_1 = -\Delta e_3 = \Delta T/2G_0, \ \Delta e_2 = 0, \tag{1.2}$$

which are the associated flow law under the condition T = const since $\Delta e_i = h(T)(\partial T/\partial\sigma_i)\Delta T$ (i = 1, 2, 3), where $h(T) = 1/G_0(T)$.

If the principal stress axes are fixed (quasisimple loading [2]), then (1.2) can be integrated

$$e_1 = -e_3 = (1/2)\gamma_0(T), \ e_2 = 0.$$

In the complete plasticity state, when T and $T_{12} > \tau_s$, $T_{23} < \tau_s$, on the basis of the mechanical model of a material it is possible to write

$$\Delta\gamma^\nu = \Delta T/G_0, \ \Delta\gamma_{12}^\nu = \Delta T_{12}/G_1,$$

where $\Delta\gamma^\nu$, $\Delta\gamma_{12}^\nu$ are simple shears originating in the direction T and T_{12}, and $G_1 = G_0(T_{12})$.

Going over to the principal axes of the stress tensor, we obtain the relations

$$\Delta e_1 = \Delta T/2G_0 + \Delta T_{12}/2G_1, \ \Delta e_2 = -\Delta T_{12}/2G_1, \ \Delta e_3 = -\Delta T/2G_0, \tag{1.3}$$

which are also represented in the form of a flow law [5]:

$$\Delta e_i = h(T)(\partial T/\partial\sigma_i)\Delta T + h_1(T_{12})(\partial T_{12}/\partial\sigma_i)\Delta T_{12} \ (i = 1, 2, 3),$$

where $h_1(T_{12}) = 1/G_0(T_{12})$.

If complete indeterminacy in the selection of the loading surface and the hardening function remains in this flow law proposed for singular loading surfaces, then (1.2) and (1.3) (the mechanical model of a material) eliminate this indeterminacy.

For quasisimple loading with $\Delta T, \Delta T_{12} \geqslant 0$, the relations (1.3) can be integrated

$$e_1 = (1/2)\gamma_0(T) + (1/2)\gamma_0(T_{12}), \ e_2 = -(1/2)\gamma_0(T_{12}), \ e_3 = -(1/2)\gamma_0(T).$$

As an example, let us consider uniaxial tension $\sigma_1 > 0$, $\sigma_2 = \sigma_3 = 0$, $T = T_{12}$, $T_{23} = 0$, then $e_1 = \gamma_0(T)$, $e_2 = e_3 = -(1/2)\gamma_0(T)$.

Now we turn to the traditional deviator plane. The projections of the principal stresses in this plane are denoted by 1, 2, 3 (Fig. 2).

On the basis of the relationships (1.2) and (1.3), and the material technical specification introduced (see Fig. 1), we conclude that hardening develops as follows on the deviator plane (Fig. 2): The loading point displaces piecewise-linear sections of the Tresk surface parallel to themselves by removing them from the origin. This fact can be obtained from the hardening scheme [6] applied to the Tresk–Saint-Venant plasticity condition.

If a thin-walled cylindrical specimen is subjected to loading by a change in the internal pressure and axial force in the initial stage of the loading, then complex loading

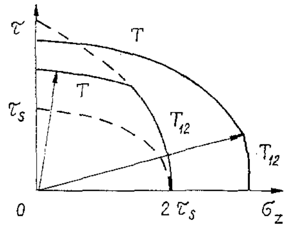

Fig. 3

can be realized by adding a torque. If torque loading occurs at the beginning, then the subsequent complex loading is realized by adding axial force and a change in internal pressure.

For simplicity, we consider a complex loading which is realized in tests on thin-walled tubular specimens in the absence of internal pressure. Then the stress state is characterized at each point of the material by the tensor

$$\begin{Vmatrix} \sigma_z & 0 & \tau_{z\theta} \\ 0 & 0 & 0 \\ \tau_{z\theta} & 0 & 0 \end{Vmatrix}$$

in the fixed coordinate system z, r, θ, where the z axis is along the generatrix, r has the radial direction, and θ the tangential.

The principal tangential stresses take the values

$$T = (1/2)\sigma_z \cos 2\varphi + \tau \sin 2\varphi,\ \ T_{12} = (1/4)\sigma_z + (1/2)T,\ \ T_{23} = -(1/4)\sigma_z + (1/2)T,$$

and their increments are

$$\Delta T = (1/2)\Delta\sigma_z \cos 2\varphi + \Delta\tau \sin 2\varphi,\ \ \Delta T_{12} = (1/4)\Delta\sigma_z + (1/2)\Delta T,\ \ \Delta T_{23} = -(1/4)\Delta\sigma_z + (1/2)\Delta T,$$

where $\tau = \tau_{z\theta}$; $\tan 2\varphi = 2\tau/\sigma_z$; φ is the angle between the directions of z and x_1 axes.

We shall consider the assumptions of the mechanical model of the material to be valid under complex loading. In this case the directions of the slip system will be rotated during loading.

For the state of incomplete plasticity, as before, the relationships (1.2) are valid in the T direction, and which take the following form in the fixed coordinate system

$$\Delta e_z = -\Delta e_\theta = (\Delta T/2G_0)\cos 2\varphi,\ \Delta\gamma^p_{z\theta} = (\Delta T/G_0)\sin 2\varphi,\ \Delta e_r = 0 \qquad (1.4)$$

and can be written in the form of the flow law

$$\Delta e_z = -\Delta e_\theta = h(T)(\partial T/\partial\sigma_z)\Delta T,\ \Delta\gamma^p_{z\theta} = h(T)(\partial T/\partial\tau)\Delta T,\ \Delta e_r = 0,$$

where $h(T) = 1/G_0(T)$.

Total unloading sets in for $\Delta T \leqslant 0$ and loading with $\Delta T = 0$ is neutral and does not result in the growth of plastic deformation.

In the state of complete plasticity with T and $T_{12} > \tau_s$, $T_{23} < \tau_s$, the relations (1.3) are valid, which take the following form in the z, r, θ coordinate system

$$\begin{aligned} \Delta e_z &= (\Delta T/2G_0)\cos 2\varphi + (\Delta T_{12}/4G_1)(1 + \cos 2\varphi), \\ \Delta e_\theta &= -(\Delta T/2G_0)\cos 2\varphi + (\Delta T_{12}/4G_1)(1 - \cos 2\varphi), \\ \Delta\gamma^p_{z\theta} &= (\Delta T/G_0 + \Delta T_{12}/2G_1)\sin 2\varphi,\ \Delta e_r = -\Delta e_z - \Delta e_\theta. \end{aligned} \qquad (1.5)$$

For the plane state of stress $\sigma_r = \tau_{rz} = \tau_{r\theta} = 0$, and the principal tangential stresses T and T_{12} are written in the form

$$T = (1/2)\sqrt{(\sigma_z - \sigma_\theta)^2 + 4\tau^2},\ \ T_{12} = (\sigma_z + \sigma_\theta)/4 + (1/2)T,$$

then for σ_θ the relationships (1.5) are representable in the form of a flow law with a singular loading surface in the general case

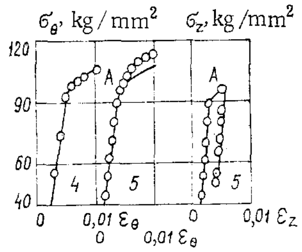

Fig. 4

$$\Delta e_z = h(T)\frac{\partial T}{\partial \sigma_z}\Delta T + h_1(T_{12})\frac{\partial T_{12}}{\partial \sigma_z}\Delta T_{12},$$

$$\Delta e_\theta = h(T)\frac{\partial T}{\partial \sigma_\theta}\Delta T + h_1(T_{12})\frac{\partial T_{12}}{\partial \sigma_\theta}\Delta T_{12},$$

$$\Delta \gamma^p_{z\theta} = h(T)\frac{\partial T}{\partial \tau}\Delta T + h_1(T_{12})\frac{\partial T_{12}}{\partial \tau}\Delta T_{12},$$

where $h_1(T_{12}) = 1/G_0(T_{12})$.

For partial unloading in the direction T_{12}, i.e., $\Delta T_{12} \leqslant 0$, $\Delta T > 0$, the relationships (1.5) go over continuously into (1.4). Total unloading sets in for ΔT, $\Delta T_{12} \leqslant 0$, and the singularity at the loading point is formed by the intersection of the surfaces T and T_{12} = const.

The condition $T = k_1$ in the stress plane σ_z, τ is a Tresk ellipse $(\sigma_z/2k_1)^2 + (\tau/k_1)^2 = 1$, while the condition $T_{12} = k_2$ is a parabola $2k_2\sigma_z + \tau^2 = 4k_2^2$, where k_1 and k_2 are arbitrary constants.

The curves corresponding to $k_1 = k_2 = \tau_s$ are displayed in Fig. 3 by dashes.

On the basis of the mechanical model of a material, we conclude that hardening develops as follows (Fig. 3) in the plane under consideration: The loading point displaces piecewise-smooth sections of the curves of constant principal tangential stresses T and T_{12} parallel to themselves by withdrawing them from the origin.

For the flow law proposed, loadings with constant value of the maximum tangential stress are neutral in the incomplete plasticity state, however, experimental investigations [7] have shown that a growth in plastic deformation occurs under such loadings. This fact is successfully described by remaining within the framework of the principal tangential stress conception [8], here the mechanical model of a material is taken as basis as is also the assumption about orthotropy of the plastic state in the form [3].

2. We present a comparison between the results of a computation using the relations proposed and test data [1, 9].

Test data are presented in [1] for thin tubular specimens of steel 30KhN3A which were loaded by internal pressure and a tensile force. For certain specimens the ratio $k = \sigma_2/\sigma_\theta$ remained constant during the test. A characteristic feature of these tests was the fact that different kinds of partial unloadings, when stress growth occurs in some directions and unloading in others were produced at definite stages of the loading for the majority of them.

The hardening modulus $G_0(T)$ was determined from the curve $\sigma_\theta(\varepsilon_\theta)$ for the specimen 4, which was loaded by pure torsion for $k = 0.5$ (Fig. 4). Results of testing specimen 5 (open circles) are also presented in Fig. 4. Up to the point A this specimen was loaded for $k = 1$, and then the stress intensity was maintained constant so that $\Delta\sigma_\theta > 0$, $\Delta\sigma_z < 0$. The results of a computation for the second loading stage are superposed by solid lines.

Data of testing thin-walled cylinders of the aluminum alloy 14S-T4 under complex loading conditions are represented in [9]. The specimens were brought into the plastic state by axial compression. The complex loading was accomplished by adding the torque $\tau_{z\theta} = \tau$.

At the time of spinup application the compressive force changed in a different manner so that the ratio is $d\sigma_z/d\tau$ = const for each of the specimens, but changes substantially from specimen to specimen.

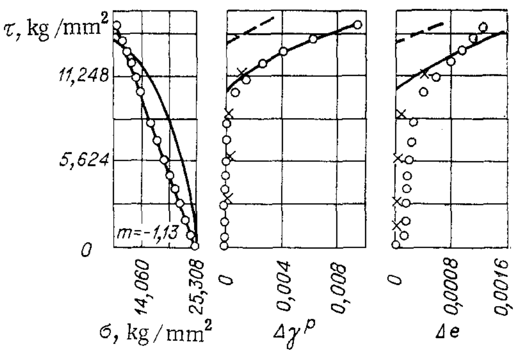

Fig. 5

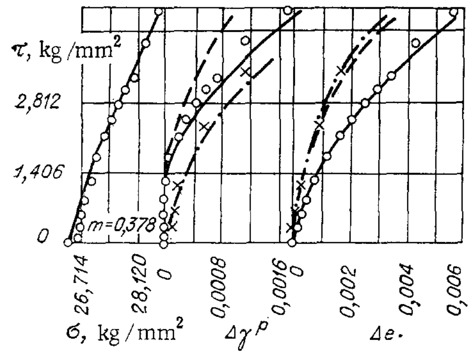

Fig. 6

The results of the tests [9] showed that the material is deformed elastically in the plane of action τ at the initial time of spinup (the initial shear modulus equals the elastic modulus). This fact is in agreement with the assumptions of the mechanical model of a material.

The principal tangential stresses T, T_{23} take the following form in the complex loading case under consideration

$$T = (1/2)\sigma\cos 2\varphi + \tau\sin 2\varphi,\ T_{23} = (1/4)\sigma + (1/2)T,$$

and their increments are

$$\Delta T = (1/2)\Delta\sigma\cos 2\varphi + \Delta\tau\sin 2\varphi,\ \Delta T_{23} = (1/4)\Delta\sigma + (1/2)\Delta T,$$

where $\sigma = -\sigma_z > 0$; $\tan 2 = 2\tau/\sigma$; φ is the angle between the directions of the θ and x_1 axes.

A state of complete plasticity is realized in the directions T and T_{23} at the initial instant of complex loading since $T = T_{23}$ and $T_{12} = 0$.

The governing relations for different kinds of spin-up from the state under consideration have the form

$$\Delta e = \frac{1}{2}\frac{\Delta T}{G_0}\cos 2\varphi + \frac{1}{4}\frac{\Delta T_{23}}{G_2}(1 + \cos 2\varphi),$$
$$\Delta\gamma^p = \left(\frac{\Delta T}{G_0} + \frac{\Delta T_{23}}{2G_2}\right)\sin 2\varphi, \tag{2.1}$$

where $\Delta e = -\Delta\varepsilon_z^p$, $\Delta\gamma^p = \Delta\gamma_{z\theta}^p$, $G_2 = G_0(T_{23})$.

The elastic constants of the aluminum alloy 14S-T4 are the following: $E = 7381.5$ kg/mm², and $\mu = 2776.85$ kg/mm². The yield point under uniaxial compression is $\sigma_s = 17.575$ kg/mm².

Results of processing the curves $\sigma_z(\varepsilon_z)$ for each of the specimens showed that cylinders loaded for $d\sigma/d\tau = m = -1.13$ and $m = 0.378$ had approximately the identical modulus $G_0(T)$ on the hardening section. For these specimens the plastic hardening modulus was selected from the curve $\gamma^p(\tau)$ for data from the experiment with $m = -1.13$.

Theoretical curves were constructed for complex loading sections by means of formulas (2.1) and are displayed in Figs. 5 and 6 by solid lines. The test data are denoted by open circles. Indicated here are the loading programs $\sigma(\tau)$ and the results of computations by other theories. The computational dependences by the Hencke—Nadai—Il'yushin deformation theory are denoted by the dash-dot lines, by flow theory with an isotropically broadening Mise-surface by dashes, and by the Batdorff—Budiansky theory by crosses.

The author is grateful to E. I. Shemyakin and V. M. Zhigalkin for useful discussions during execution of this research.

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STATIONARY CONFIGURATION OF FIBERS FORMED UNDER NONISOTHERMAL CONDITIONS

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UDC 532.222 + 681.7.068.4

One of the important problems of chemical technology is fiber molding. Nevertheless, the substantial influence of heat transfer on fiber characteristics has been investigated insufficiently. The first step is to obtain stationary solutions. The stationary fiber configurations are computed numerically in [1]. In this paper analytic solutions of the stationary problem are obtained under the assumption of large activation energy of the viscous flow.

We shall consider the melt to be shaped to be a Newtonian fluid with viscosity dependent on the temperature according to the Arrhenius law. Such high values of the viscosity correspond to sufficiently low temperatures that flow practically ceases and the material solidifies. This approximation corresponds best to the behavior of melted glass [2, 3].

Let us examine the two most widespread technological processes: 1) drawing a fiber from a cylindrical glass blank heated to high temperature (Fig. 1a), and 2) drawing through a spinneret hole from a tank containing the melt (Fig. 1b). In both cases the fiber being drawn cools and solidifies during motion in the air. In the situations under consideration we shall consider the material to advance at a constant given velocity V_0. At the end of the shaping section, the fiber is incident on a receiving unit (bobbin) giving a certain value of the longitudinal velocity. We shall conduct the description within the framework of quasi-one-dimensional equations of continuity, momentum [4, 5], and heat propagation by assuming the flow to change sufficiently slowly along the fiber:

$$\begin{gathered}\partial f/\partial t + \partial fV/\partial x = 0,\ f = \pi a^2,\\ \rho f(\partial V/\partial t + V\partial V/\partial x) = \partial P/\partial x,\ P = 3\mu f\partial V/\partial x,\ \mu = \mu_0 \exp(U/RT),\\ \rho f c\left(\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}\left(\lambda f\frac{\partial T}{\partial x}\right) - 2\pi a q_w.\end{gathered} \tag{1}$$

Here t is the time, x is the coordinate measured along the fiber axis, f is the area of the fiber section (it is considered that it has a circular section of radius a), V is the magnitude of the axial velocity in the fiber, T is the temperature, ρ, μ, c, λ are the density, viscosity, specific heat, and heat conduction of the melt, P is the magnitude of the axial force in the fiber section, μ_0 and U are the preexponential factor and the activation energy

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 139-144, November-December, 1982. Original article submitted October 26, 1981.

0021-8944/82/2306-0865$07.50